Measure Theory and Ergodic Theory

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The foundational knowledge relating to Measure & Ergodic Theory, that has not been covered elsewhere, was provided by Robertson (2023).

Definition 0.1 (Gleason (2010))**.** A *topological measure space* $(X, \mathscr{B}, \mu)$ is a topological space (X, τ) such that $\mathscr{B}$ is generated by the open sets defined by the topology τ, i.e., $\mathscr{B} = \text{Borel}(X) = \sigma(\tau)$, and μ is a measure on this space.

A *Borel measure* is the measure μ on a topological measure space $(X, \mathscr{B}, \mu)$ where (X, τ) is Hausdorff.

A *regular (Borel) measure* is a measure on a Borel measure space $(X, \mathscr{B}, \mu)$ such that the following hold:

1. *Finite Compact Measure*: For any compact subset $K \subseteq X$, then $\mu(K) < \infty$.
2. *Outer Regularity*: For any $B \in \mathscr{B}$, then

$$\mu(B) = \inf\{\mu(C) \mid B \subseteq C, C \text{ is open}\}.$$

3. *Inner Regularity*: For any $U \in \tau$, or, in other words, any open subset $U \subseteq X$, then

$$\mu(U) = \sup\{\mu(K) \mid K \subseteq U, K \text{ compact}\}.$$

A *left-Haar measure* [or *right-Haar measure*] on a topological group (Γ, τ) is a non-zero regular Borel measure μ on Γ such that $\mu(\gamma \cdot B) = \mu(B)$ [or $\mu(B \cdot \gamma) = \mu(B)$] for all $\gamma \in \Gamma$ and $B \in \sigma(\tau)$.

Outlinks

- Density
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- Actions
- Factor Maps
- Furstenberg's Correspondence Principle
- A Short Proof of a Generalised Conjecture of Erdős for Amenable Groups
- Recurrence and Ergodic Theorems

Robertson, D. (2023). '*MATH41021/61021 measure and ergodic theory*', Available at: https://personalpages.manchester.ac.uk/staff/donald.robertson/teaching/23-24/41021 (Accessed: 22 January 2024).

Gleason, J. (2010). '*Existence and uniqueness of haar measure*',.