Kronecker Factor

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Definition 0.1 (Jamneshan and Kreidler, 2025, Definition 7.1.14 & Subsection 13.3)**.** Let (X, T) be a measure-preserving system. We then write $J_{\text{kro}} : (X_{\text{kro}}, T_{\text{kro}}) \to (X, T)$ for the extension $J_E : (X_E, T_E) \to (X, T)$ defined by the invariant Markov sublattice $E = \mathrm{L}^2(X)_{\text{ds}}$ and call $(X_{\text{kro}}, T_{\text{kro}})$ the **Kronecker subsystem** of (X, T).

We refer to $(X_{\text{kro}}, \tau_{\text{kro}})$ as the *Kronecker factor* of the system (X, τ).

Proposition 0.1 (cf. Host, 2019, Proposition 5)**.** *Let (X, Γ) be a topological dynamical system where Γ is an amenable group, $x_0 \in X$, and μ be an ergodic invariant probability measure supported on the closed orbit of x_0 under the action of Γ.*

Let (Z, m_Z, H) be the Kronecker factor *of (X, μ, Γ), with factor map $\pi : X \to Z$.*

Let $X \times Z$ be endowed with the group action of $\Gamma \times H$. Let $\tilde{\mu}$ be the measure on $X \times Z$ and image of μ under the map $X \to X \times Z$ where $x \mapsto (x, \pi(x))$.

Then there exists a Følner sequence $\tilde{\Phi}$ and a point $z_0 \in Z$ such that (x_0, z_0) is generic for $\tilde{\mu}$ along $\tilde{\Phi}$.

The sources given by Host (2019) for this result were Kra and Host (2007), Proposition 6.1 and Host and Kra (2018), Proposition 24.3.

Jamneshan, A. and Kreidler, H. (2025). '*ISem 28: Ergodic structure theory and applications*',.

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